Analysis of the trade-off between resolution and bandwidth for a nanoforce sensor based on diamagnetic springs.

Emmanuel Piat, Joël Abadie, Stéphane Oster

ICRA 2012 – St Paul, Minnesota

Context of study

Only micro-nano force effects can be directly measured

A force transducer is needed

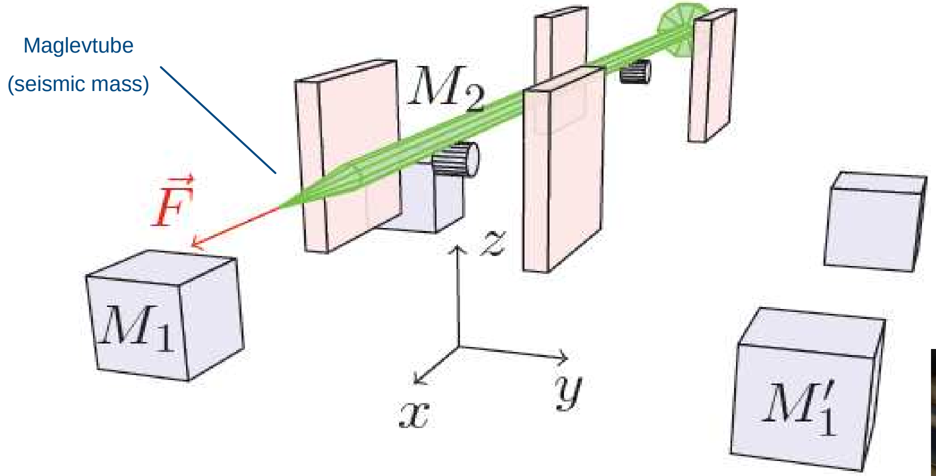
Deformation
of an elastic µstructure
when a force is applied on it

Displacement
of a rigid seismic mass
when a force is applied on it

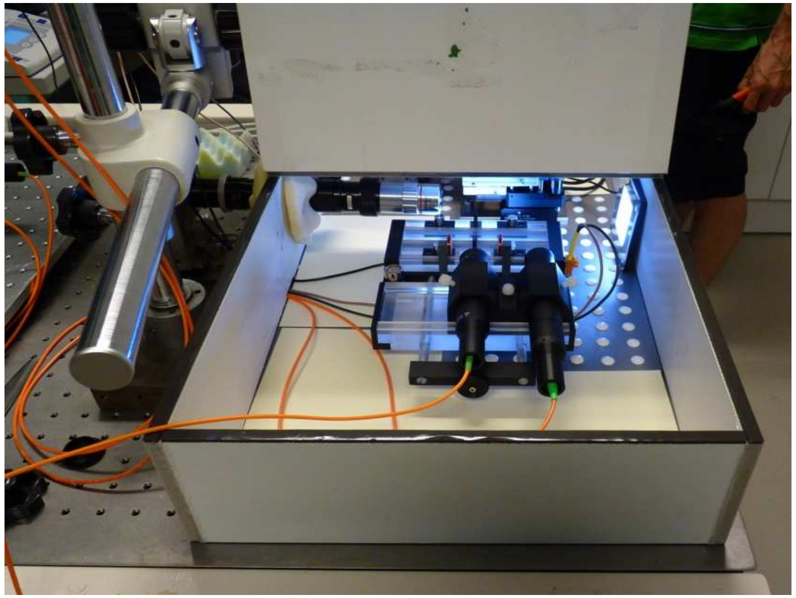
Displacement measured with appropriate sensors

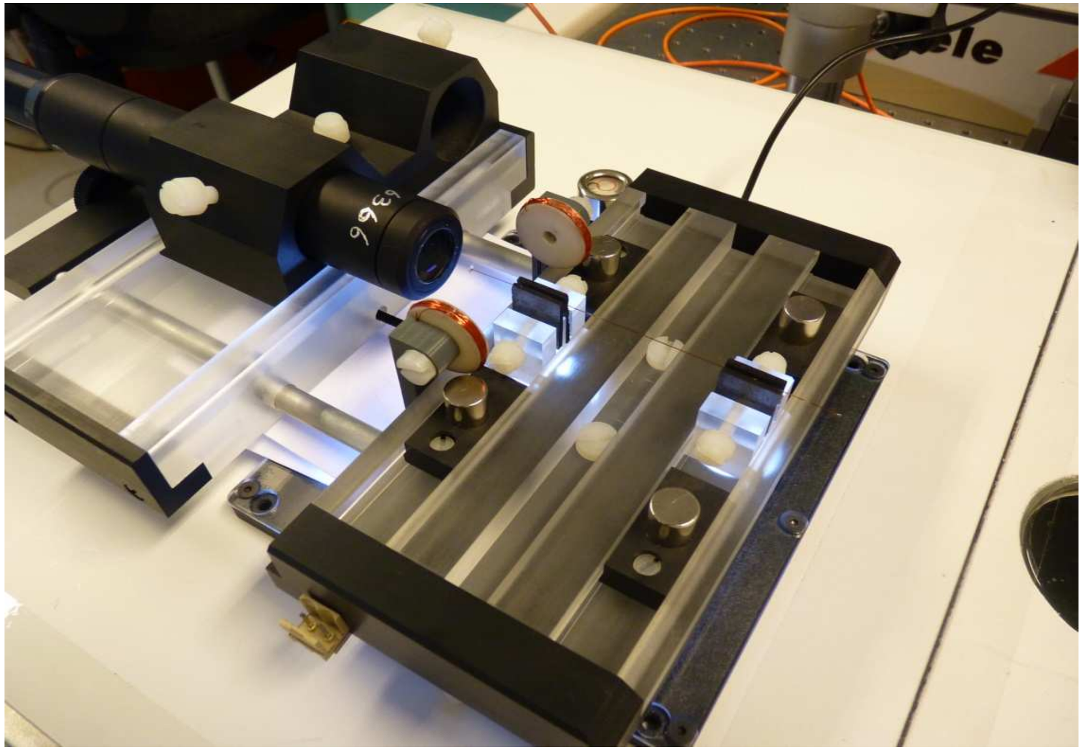
Knowing the displacement, the force must be reconstructed

Sensor configuration

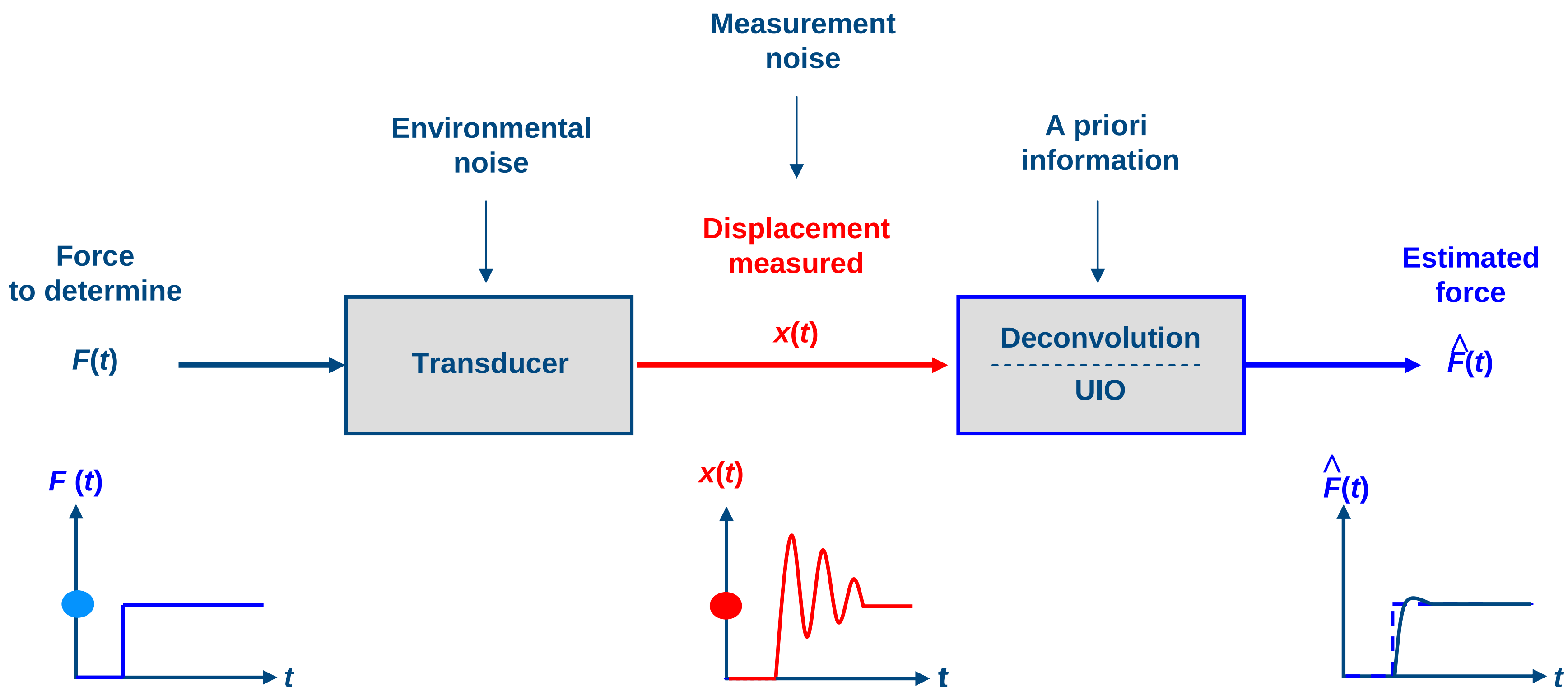


Force measured: along x axis
Stiffness: 0.005 N/m to 0.03 N/m
Typ. resolution: 1 to 5 nN
Range: 1 nN to 40 µN
Mass: 20 to 80 mg
Typ. resonant frequency: 3 Hz

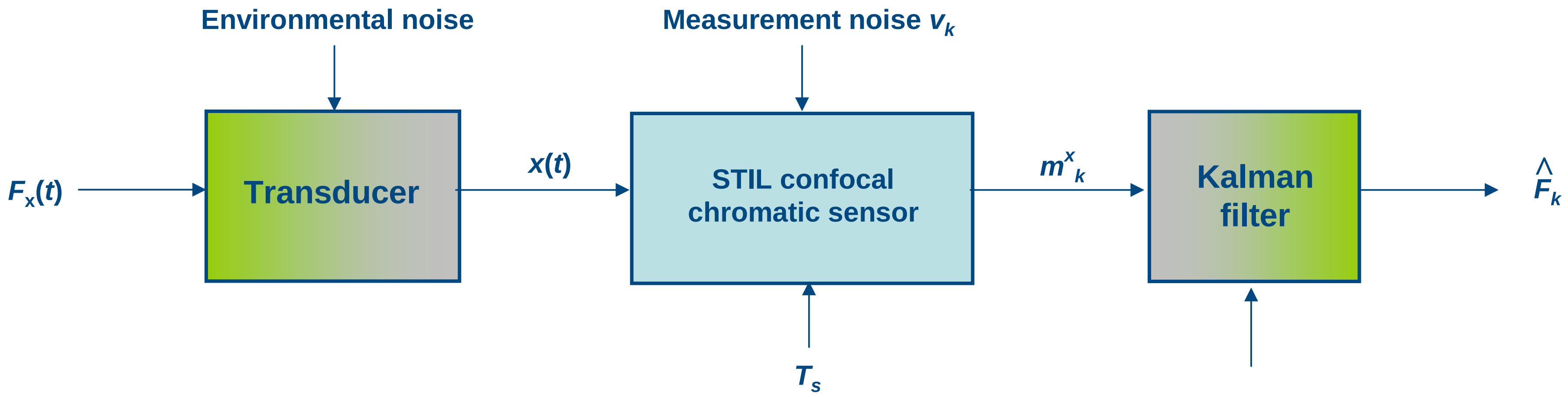




Passive force sensor



Signal processing



- Discretized uncertainty model for $F_x(t)$
- Discretized transducer model (2nd order dynamic)
- Measurement noise model

Confocal chromatic sensor (CL2 + MG140)

Zero-mean white gaussian noise v_k

Variance: $E[v_k^2] = R$

Typ. $R = 1.44 \times 10^{-16}$ m²

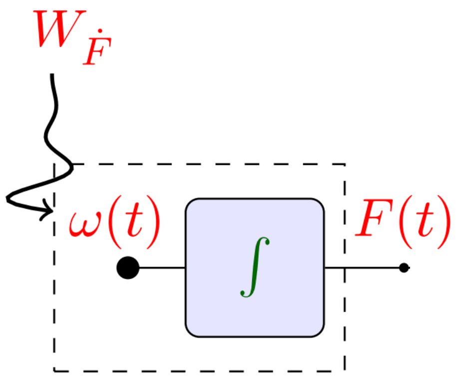
Kalman filter tuning parameter

Time-varying Kalman filter synthesis:

E. Piat, J. Abadie, S. Oster, Nanoforce estimation with Kalman filtering applied to a force sensor based on diamagnetic levitation, Proc. of the IEEE int. Conf. on Intelligent Robots and Systems (IROS), pp 39-44, San Francisco, USA, Sept. 25-30 2011.

Deconvolution of a noisy ouput: introduce a necessary trade-off between resolution and bandwidth

Driven here by a single scalar parameter (N^2/Hz): $W_{\dot{F}}$



Uncertainty modeling of the input force

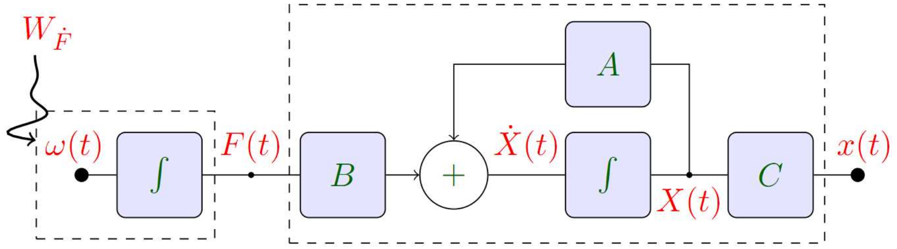
$$\dot{F}(t) = \omega(t)$$

Autocorrelation function of internal $\omega(t)$ white gaussian process:

$$\phi_{\omega,\omega}(\tau) = W_{\dot{F}}\delta(\tau) \qquad \forall \tau \in \mathbb{R}$$

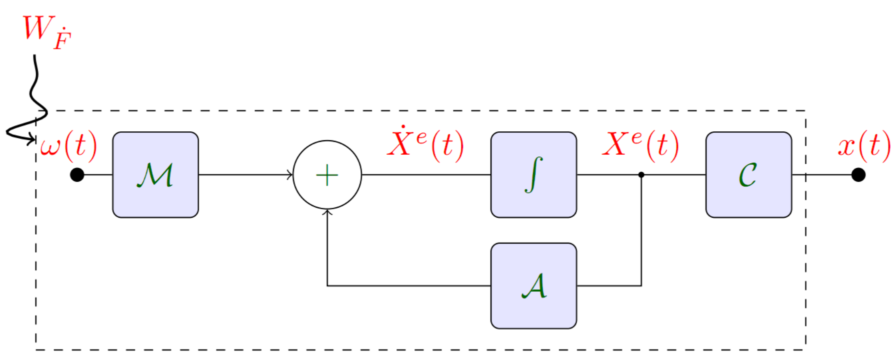
Power Spectral Density (PSD) chosen by the end-user

Force estimation



Input force model ① (uncertain)

Transductor model ② (deterministic)



Extended state-space model ③ including the uncertain modeling of the force

$$X^e(t) = \begin{bmatrix} x & \dot{x} & F \end{bmatrix}^\mathsf{T}$$

Discretization

$x(t)$ measurement: m_k^x → **Time-varying Kalman filter** →

$$X_k^e = \begin{bmatrix} x_k & \dot{x}_k & F_k \end{bmatrix}^\mathsf{T}$$

Measurement noise variance: ***R***

Ext. state ③ uncertainty cov. matrix: ***Q***

(mathematical consequence of ① + ② merging)

$$Q = W_{\dot{F}}\,\eta(T_s)$$

adjust the behavior of the estimation

Trade-off analysis

Study for fixed values of W_F and T_s (and independence of *a priori* knowledge on X_0^e)

Time-varying Kalman filter → **Steady-state Kalman filter**

A 3rd order-state equation:

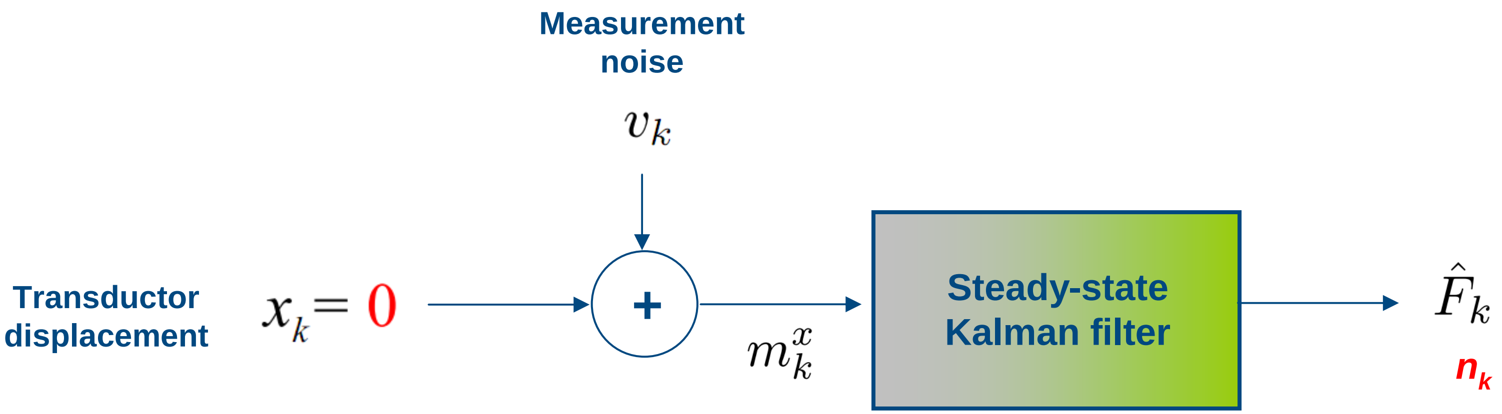
$$\hat{X}^e_{k+1|k} = A^K \hat{X}^e_{k|k-1} + B^K m^x_k$$
$$\hat{F}_k = C^K \hat{X}^e_{k|k-1} + D^K m^x_k$$

m^x_k → **Steady-state Kalman filter** → $\hat{F}_k$

A^K, B^K, C^K, D^K are functions of R and $Q = W_F\,\eta(T_s)$

Force sensor resolution study

Corresponds to the level of noise n_k in the force estimation



n_k dynamic

v_k **substituted to** m_k^x **in previous state-equation**

$$\hat{X}^e_{k+1|k} = A^K \hat{X}^e_{k|k-1} + B^K v_k$$
$$n_k = C^K \hat{X}^e_{k|k-1} + D^K v_k$$

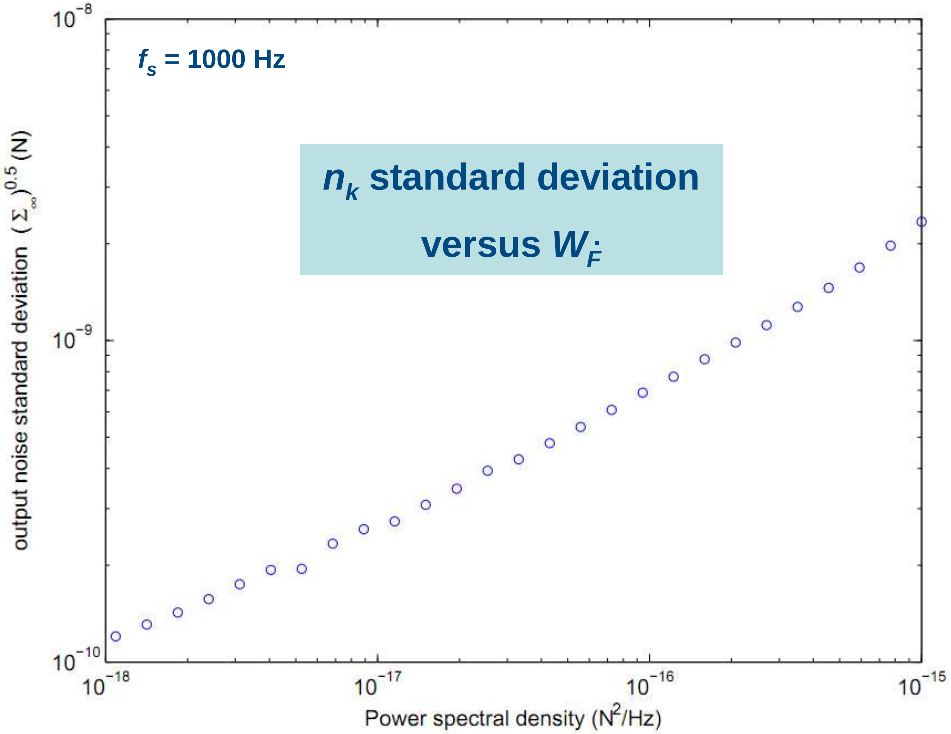
n_k statistical properties

Mean $\mu_k = 0 \quad \forall k$

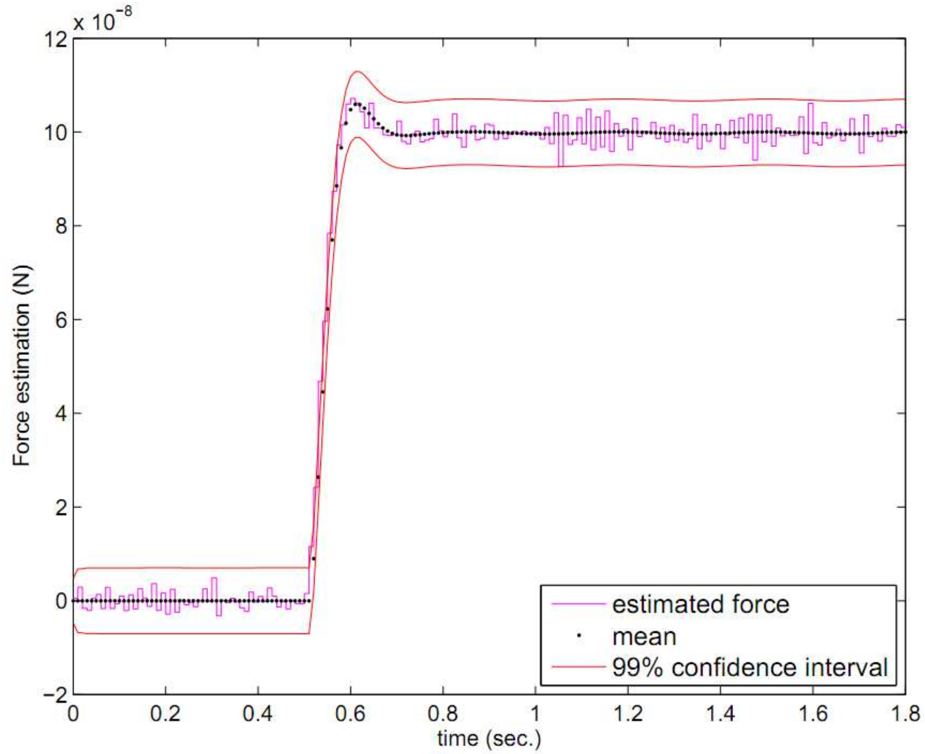
Variance $\Sigma_k = C^K S_k {C^K}^\mathsf{T} + D^K R {D^K}^\mathsf{T}$

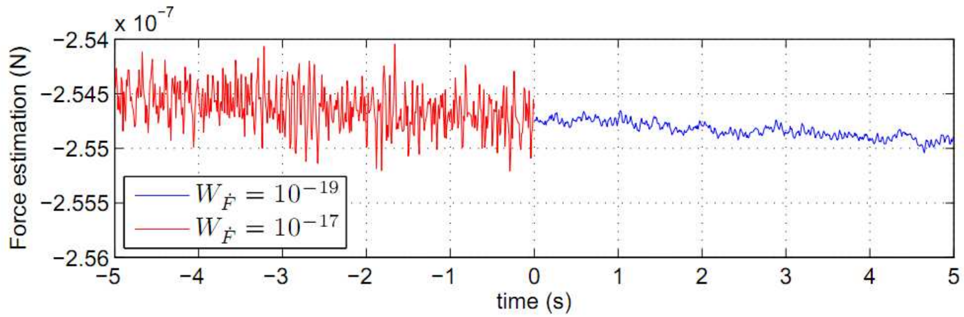
$$S_{k+1} = A^K S_k {A^K}^\mathsf{T} + B^K R {B^K}^\mathsf{T}$$

Force sensor resolution study

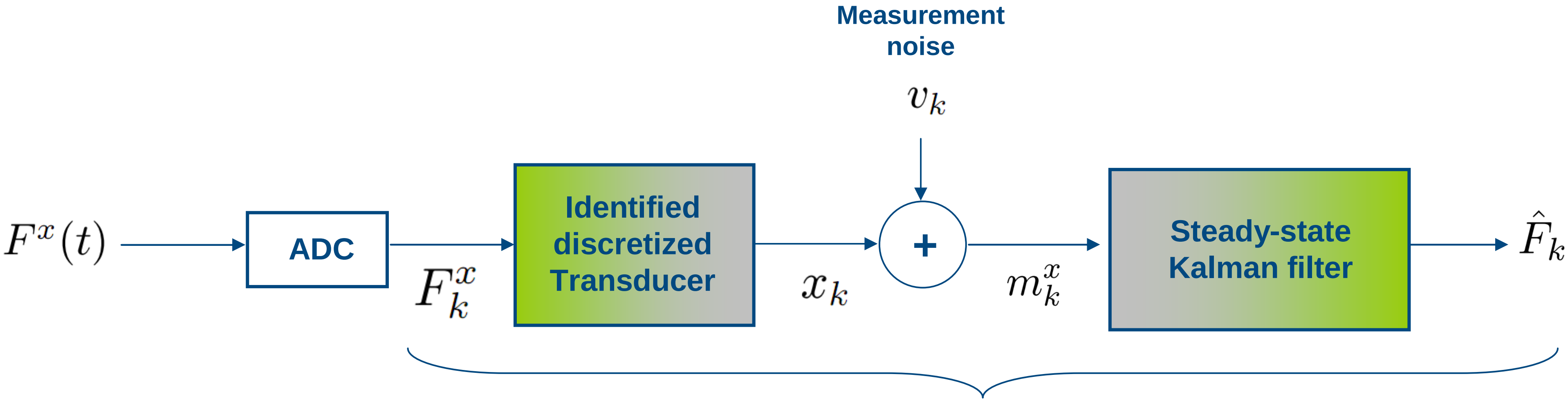


$W_{\dot{F}}$





Force sensor bandwidth study



$$\mathcal{X}_k = \begin{bmatrix} X_k \\ \hat{X}^e_{k|k-1} \end{bmatrix}$$

$$\mathcal{X}_{k+1} = A_g \mathcal{X}_k + B_g \begin{bmatrix} F_k^x \\ v_k \end{bmatrix}$$

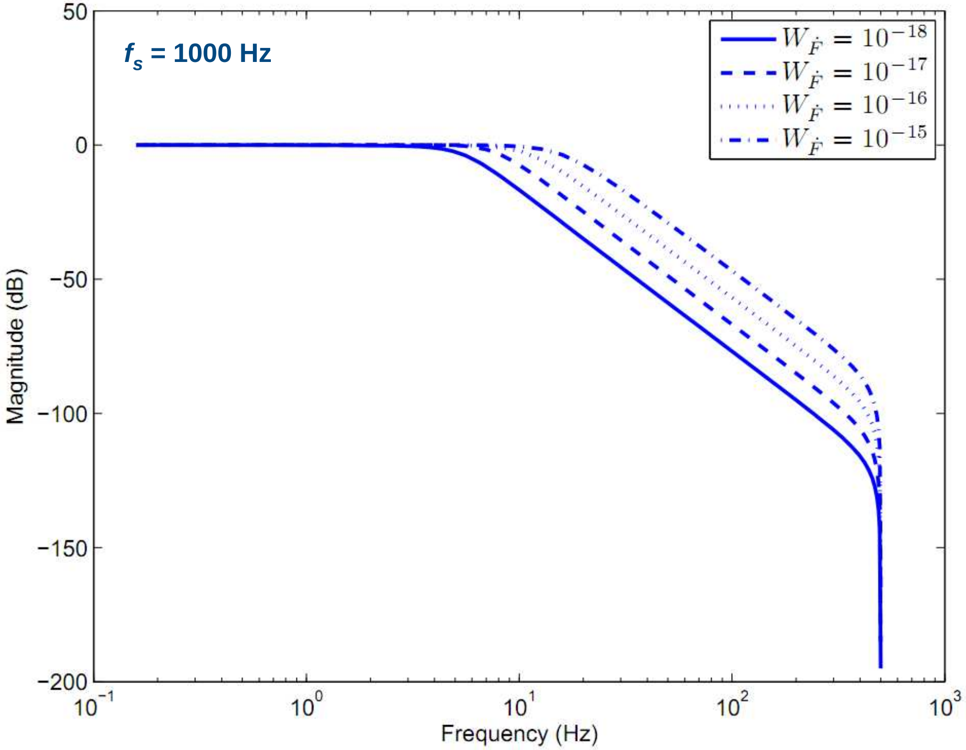
$$\hat{F}_k = C_g \mathcal{X}_k + D_g \begin{bmatrix} F_k^x \\ v_k \end{bmatrix}$$

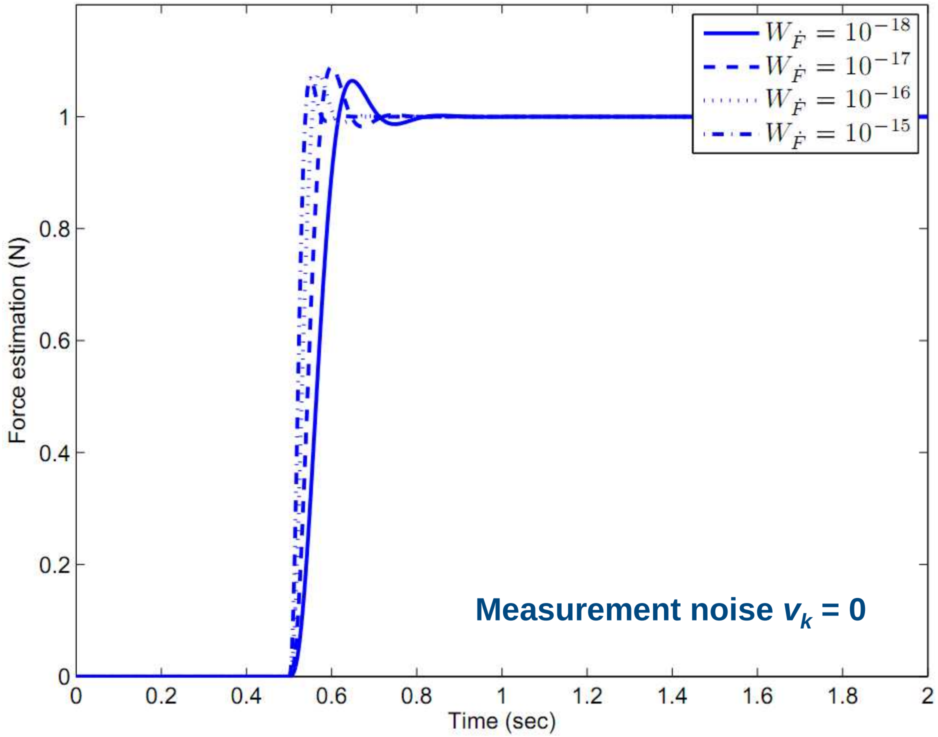
Associated transfer function

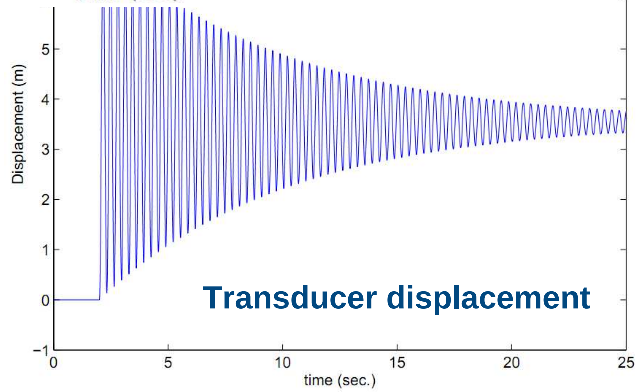
$$\frac{\hat{F}(e^{j\omega})}{F^x(e^{j\omega})}$$

with $v_k = 0$

Force sensor bandwidth study



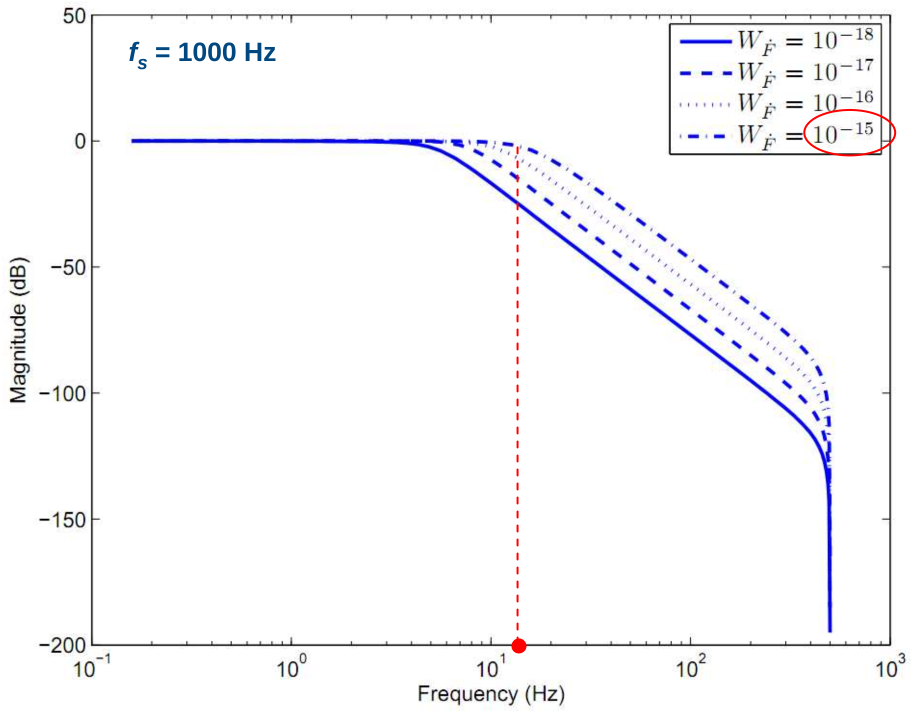


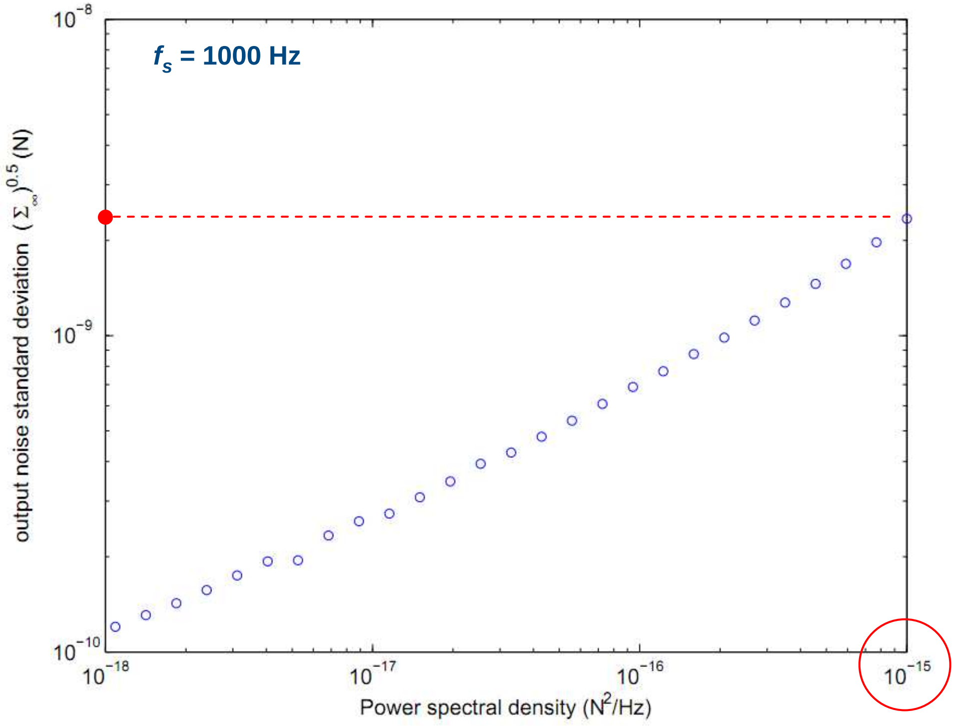


UFC UNIVERSITÉ DE FRANCHE-COMTÉ
cnrs
ensmm
utbm

$W_{\dot{F}}$ choice help

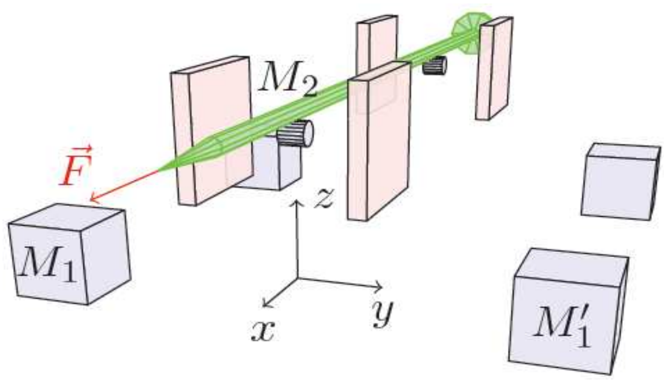
Bandwidth | For a given T_s | Resolution

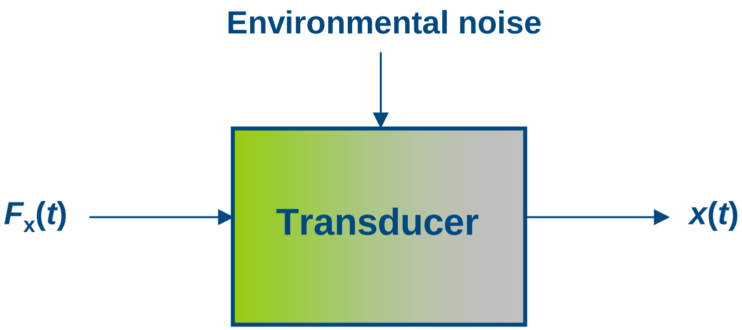




Transducer resonant frequency: 3 Hz

Conclusion





The force estimation has to take into account the behavior due to the mass inertia

Estimation processing driven by one parameter

The parameter effect on the trade-off resolution / bandwidth is fully characterized

Design Drawbacks

Open-loop design

Extreme sensitivity to external disturbing forces (seismic and subsonic vibrations, …)

In progress

New modeling including these disturbances

Future design with real-time disturbances measurement and closed-loop disturbances compensation

J. Abadie, E. Piat, S. Oster, M. Boukallel, *Modeling and experimentation of a passive low frequency nanoforce sensor based on diamagnetic levitation*, **Sensors and Actuators: A. Physical**, 2012, 173:227-237

E. Piat, J. Abadie, S. Oster, *Nanoforce estimation based on Kalman filtering and applied to a force sensor using diamagnetic levitation*, **Sensors and Actuators: A. Physical**, 2012, 179:223-236

Please visit our webpage !

www.femto-st.fr/en

AS2M department

SPECIMeN Group

SPECIMeN Group

Sensing strategies, Perception and Characterization at Micro- and Nano-scales

AS2M Dep[t] – Automatic Control and Micro-Mechatronic Systems

http://www.femto-st.fr/fr/Departements-de-recherche/AS2M/Accueil/