Nanoforce estimation with Kalman filtering applied to a force sensor based on diamagnetic levitation.

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µforce measurement

Only force effects can be directly measured

A force sensitive part (transducer) is needed

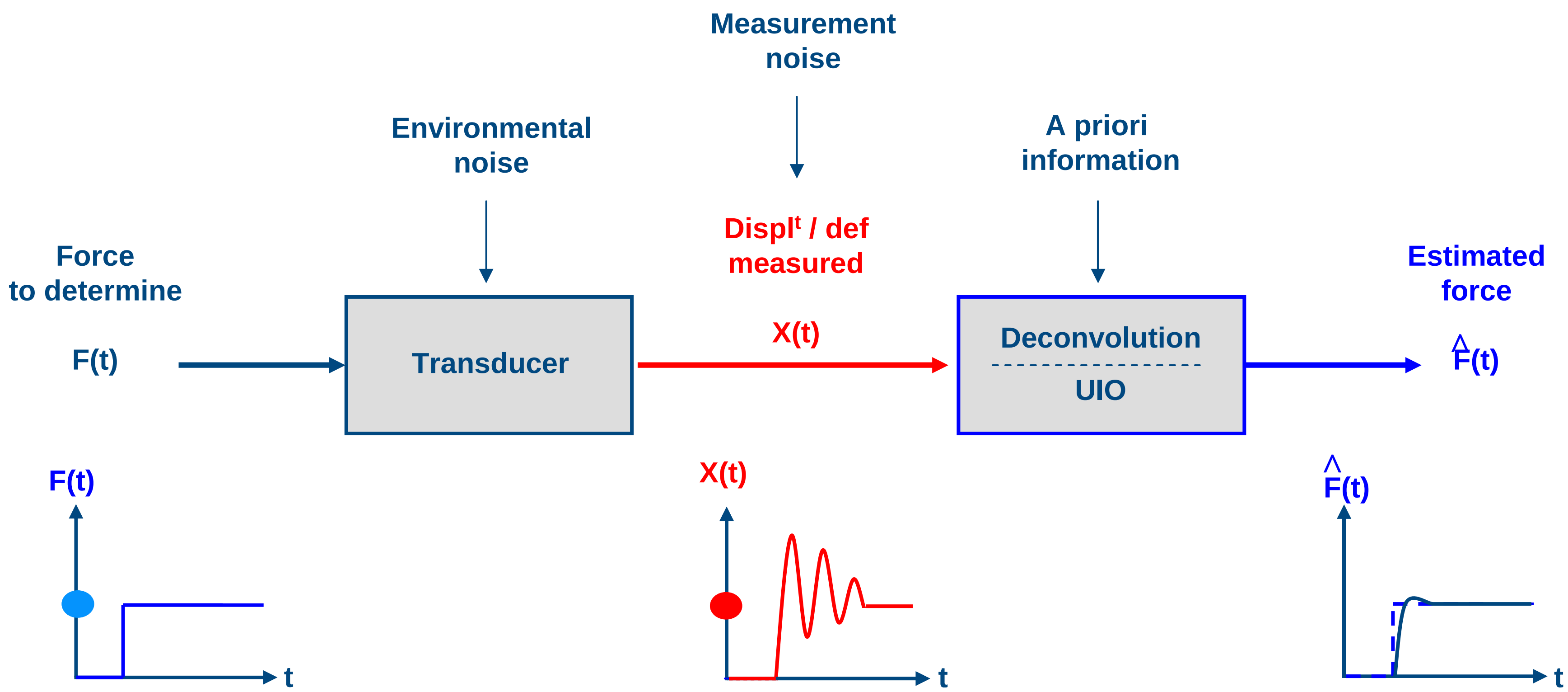
Deformation
of an elastic µstructure
when a force is applied on it

Displacement
of a rigid seismic mass
when a force is applied on it

Deformations or displacements measured with appropriate sensors

Knowing the effect, the cause (ie the force) must be reconstructed

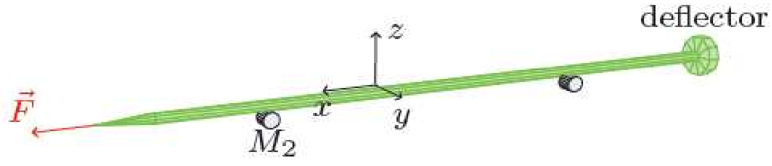
Passive µforce sensor



Positioning

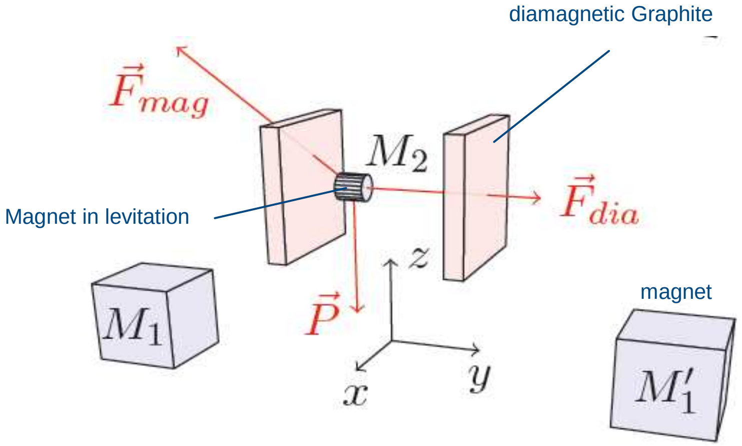
Development of a new sensor with small stiffness and a large force measurement range

Transducer = **Macroscopic seismic mass**
Naturally stable with 6 dof



Passive stability = **Done with magnetic springs**
& a repulsive physical principle

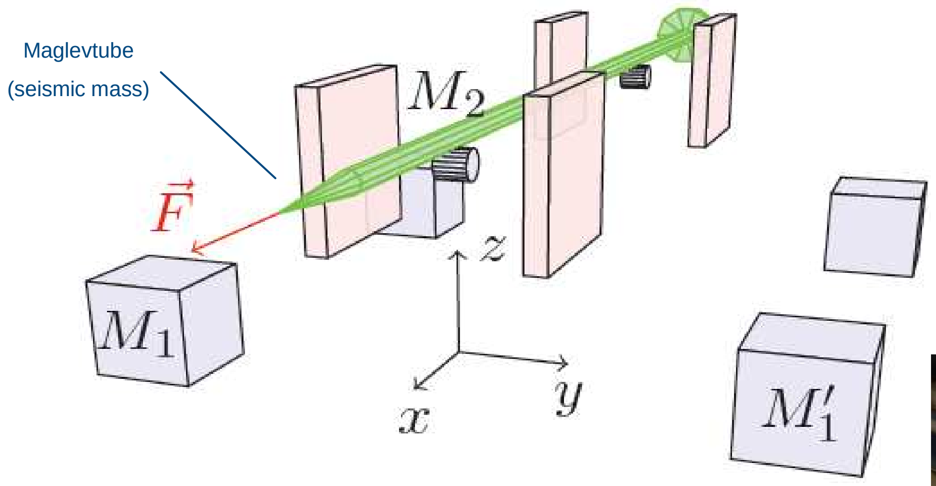
Diamagnetic effect



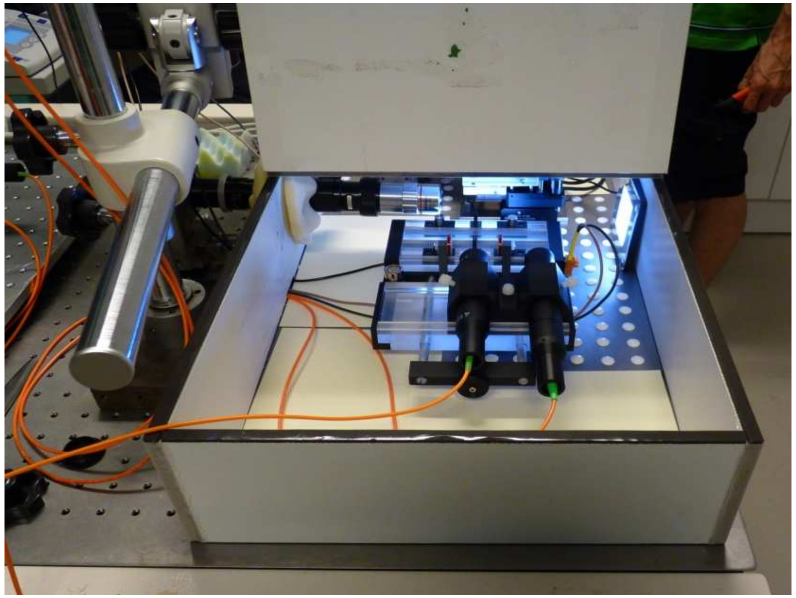
Passive force sensor with magnetic springs

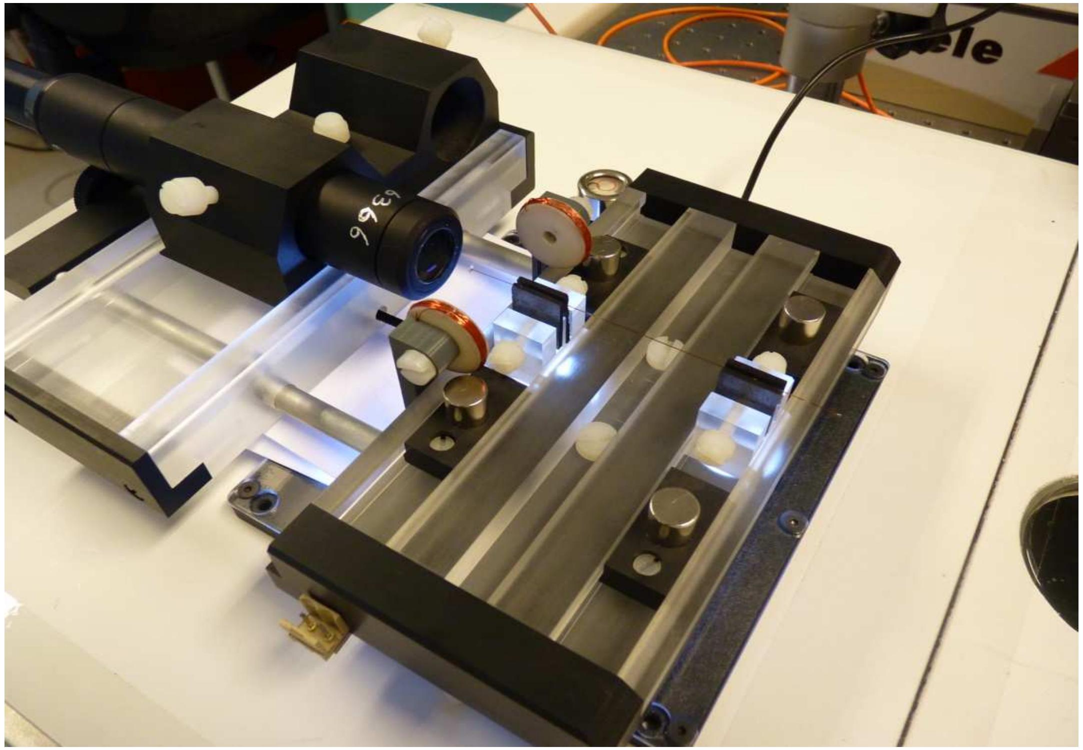
easy to produce with classical assembly approaches

Sensor configuration

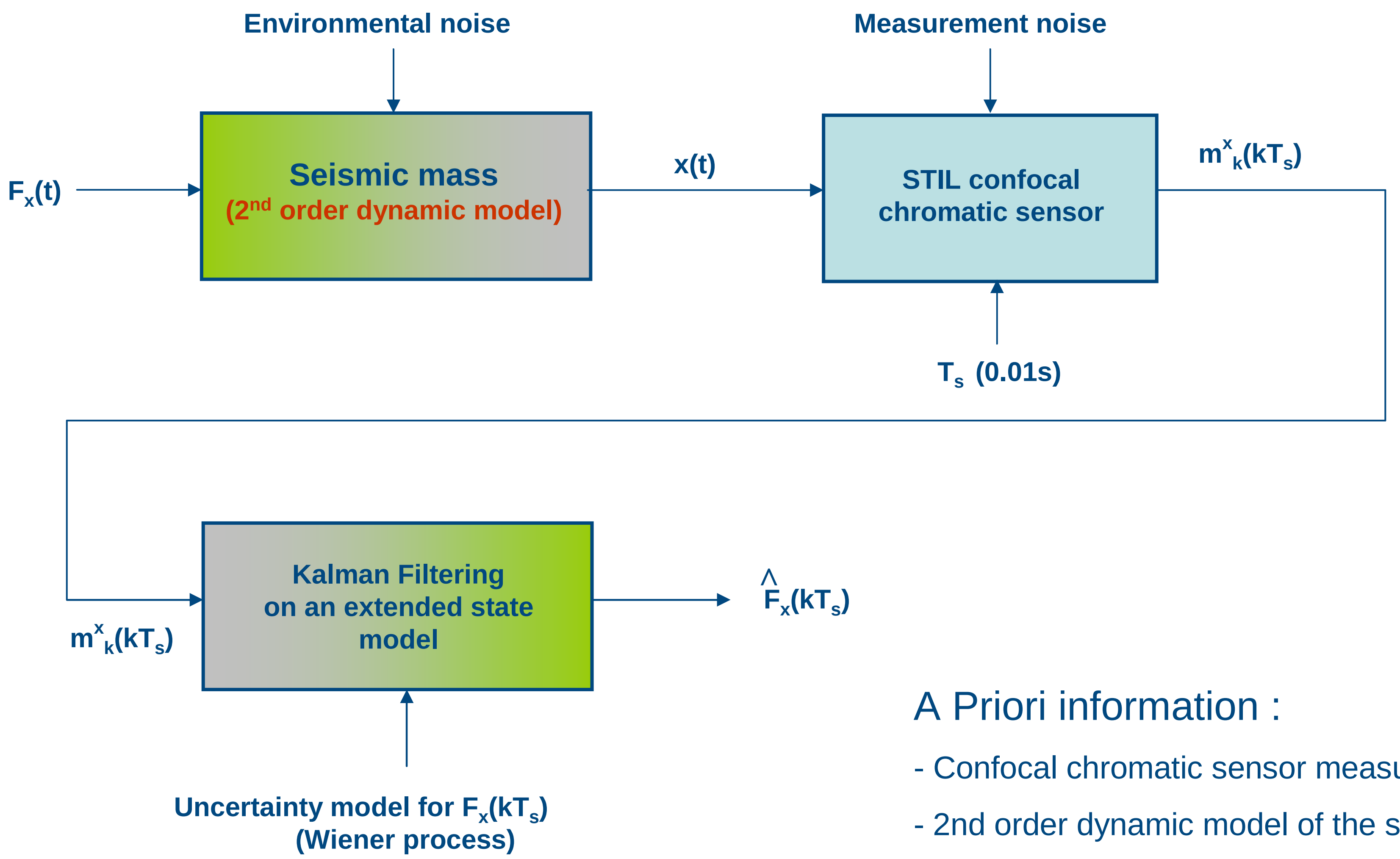


Force measured: **along x axis**
Stiffness: **0.005 N/m to 0.03 N/m**
Typ. resolution: **0.5 to 5 nN**
Range: **1 nN to 40 µN**
Mass: **20 to 80 mg**





Non steady state measurement



A Priori information :

- Confocal chromatic sensor measurement noise
- 2nd order dynamic model of the seismic mass
- Uncertainty model for the force F_x

Confocal chromatic sensor

Confocal Chromatic sensor specifications (CL2 + MG140) :

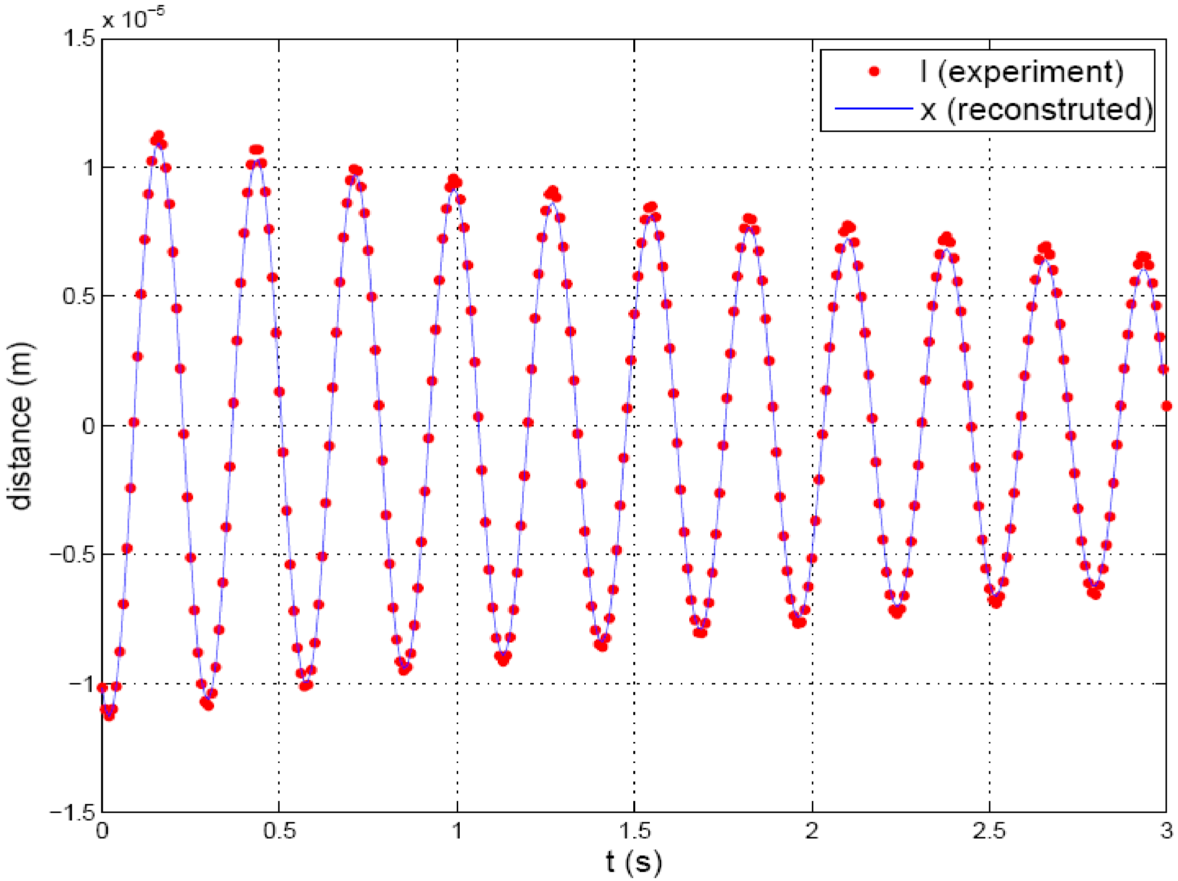
- Measuring range : 300 µm
- Working distance : 11 mm
- Axial resolution : 0.012 µm

Zero-mean white gaussian noise v_k

with $E[\mathbf{v_k}^2] = R$

$R = 1.44 \times 10^{-16}\ m^2$

Seismic mass dynamic model identification

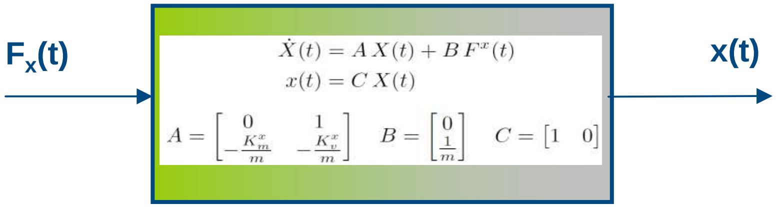


Measured by confocal sensor and reconstructed ZIR of the maglevtube displacement

Identification done using Zero Input Response (ZIR) of the maglevtube

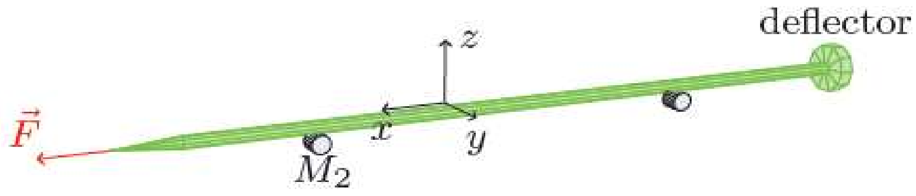
Gives the parameters of the maglevtube dynamic model, including :

- **magnetic stiffness K^x_m**
- **viscous damping coeff K^x_v**
- **maglevtube mass m**



$$\dot{X}(t) = A\,X(t) + B\,F^x(t)$$
$$x(t) = C\,X(t)$$
$$A = \begin{bmatrix} 0 & 1 \\ -\frac{K^x_m}{m} & -\frac{K^x_v}{m} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Uncertainty model of the force



Input force model based on a stochastic Wiener process :

$$\dot{F}(t) = \omega(t)$$

with $\omega(t)$ a zero-mean infinite-variance white gaussian stochastic process

Autocorrelation function of the $\omega(t)$ process is :

$$\phi_{\omega,\omega}(\tau) = W_{\dot{F}}\,\delta(\tau) \qquad \forall \tau \in \mathbb{R}$$

with $W_{\dot{F}}$ its power spectral density

Extended state-space model

Maglevtube dynamic
(2nd order dynamic model)

Force model
(stochastic Wiener process)

Extended state-space model

$$\dot{X}^e(t) = \mathcal{A}\,X^e(t) + \mathcal{M}\omega(t)$$
$$x(t) = \mathcal{C}X^e(t)$$

$$X^e(t) = \begin{bmatrix} x & \dot{x} & F \end{bmatrix}^{\mathsf{T}}$$

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{K_m^x} & -\frac{1}{K_v^x} & \frac{1}{m} \\ 0 & 0 & 0 \end{bmatrix} \quad \mathcal{M} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathcal{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Discretization of extended model

Extended state-space model

$$\dot{X}^e(t) = \mathcal{A}\,X^e(t) + \mathcal{M}\omega(t)$$
$$x(t) = \mathcal{C}X^e(t)$$

zoh discretization
(sampling period T_s)

Discrete extended state-space model

$$X^e_{k+1} = \mathcal{F}\,X^e_k + \Omega_k$$
$$x_k = \mathcal{C}\,X^e_k$$

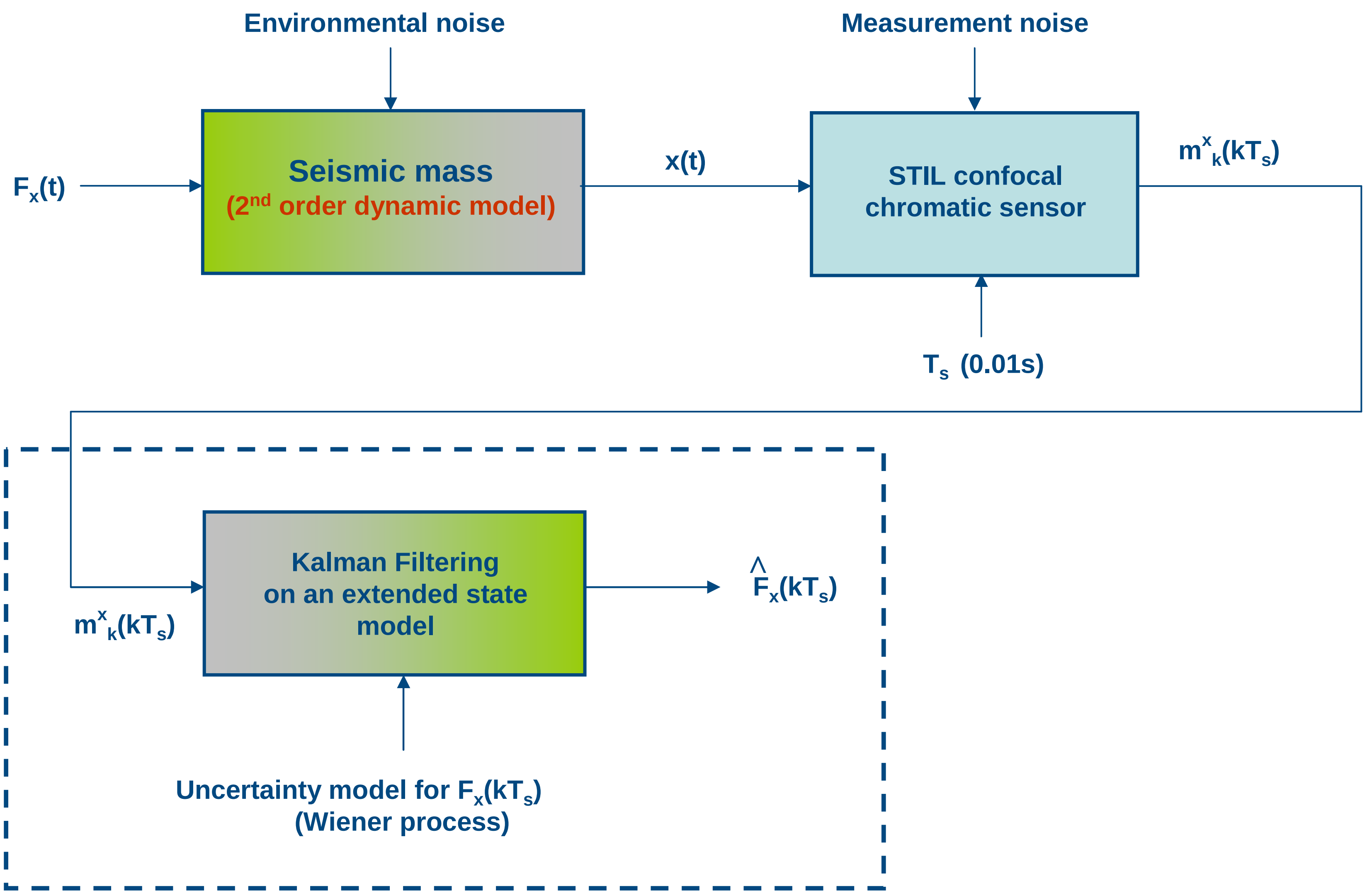
with :

$$\mathcal{F} = e^{\mathcal{A}\,T_s}$$
$$\Omega_k = \begin{bmatrix}\omega_k^x & \omega_k^{\dot{x}} & \omega_k^F\end{bmatrix}^{\mathsf{T}}$$
$$X^e_k = \begin{bmatrix}x_k & \dot{x}_k & F_k\end{bmatrix}^{\mathsf{T}}$$

and :

$$Q = E\left[\Omega_k \Omega_k^{\mathsf{T}}\right] = W_{\dot{F}}\,\eta(T_s)$$
$$\eta(T_s) = \int_0^{T_s} e^{\mathcal{A}t}\mathcal{M}\mathcal{M}^{\mathsf{T}}e^{\mathcal{A}^{\mathsf{T}}t}dt$$

Force estimation using Kalman filter



Force estimation using Kalman filter

$m^x_k(kT_s)$

$$m_k^x = x_k + v_k$$
$$E[v_k^2] = R$$

$$\hat{X}^e_{k|k-1} = \mathcal{F}\,\hat{X}^e_{k-1}$$
$$P_{k|k-1} = \mathcal{F} P_{k-1} \mathcal{F}^{\mathsf{T}} + Q$$
$$K_k = P_{k|k-1}\mathcal{C}^{\mathsf{T}}\left(\mathcal{C}P_{k|k-1}\mathcal{C}^{\mathsf{T}} + R\right)^{-1}$$
$$\hat{X}^e_k = \hat{X}^e_{k|k-1} + K_k\left(m_k^x - \mathcal{C}\hat{X}^e_{k|k-1}\right)$$
$$P_k = (I - K_k\mathcal{C})\,P_{k|k-1}$$

$\hat{F}_x(kT_s)$

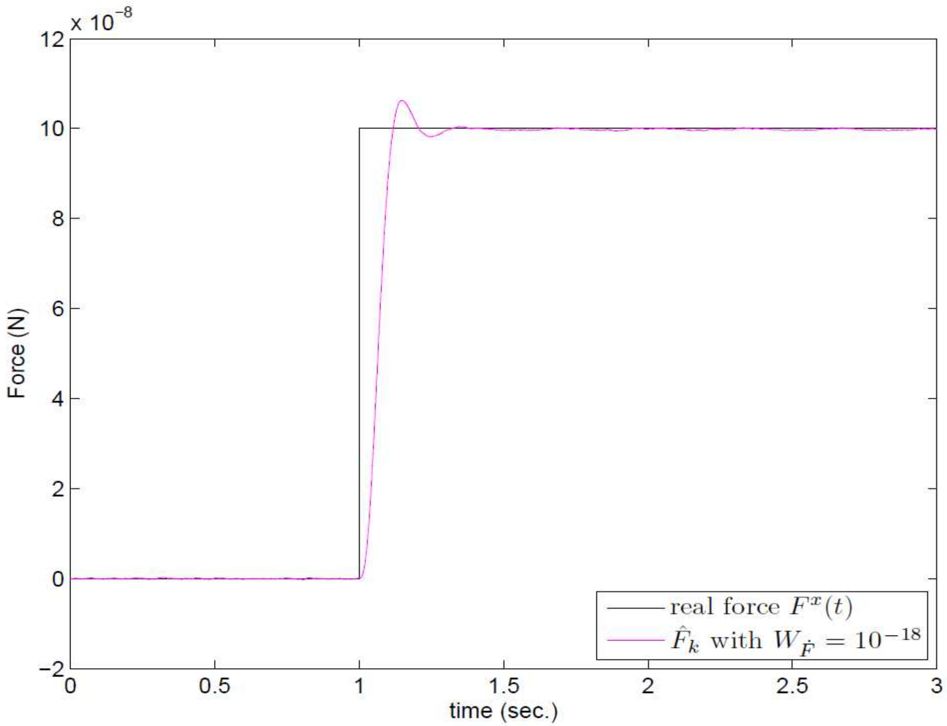
$$\hat{F}_k = \mathcal{C}_F\,\hat{X}^e_k$$
$$\mathcal{C}_F = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Uncertainty model for $F_x(kT_s)$
(Wiener process)

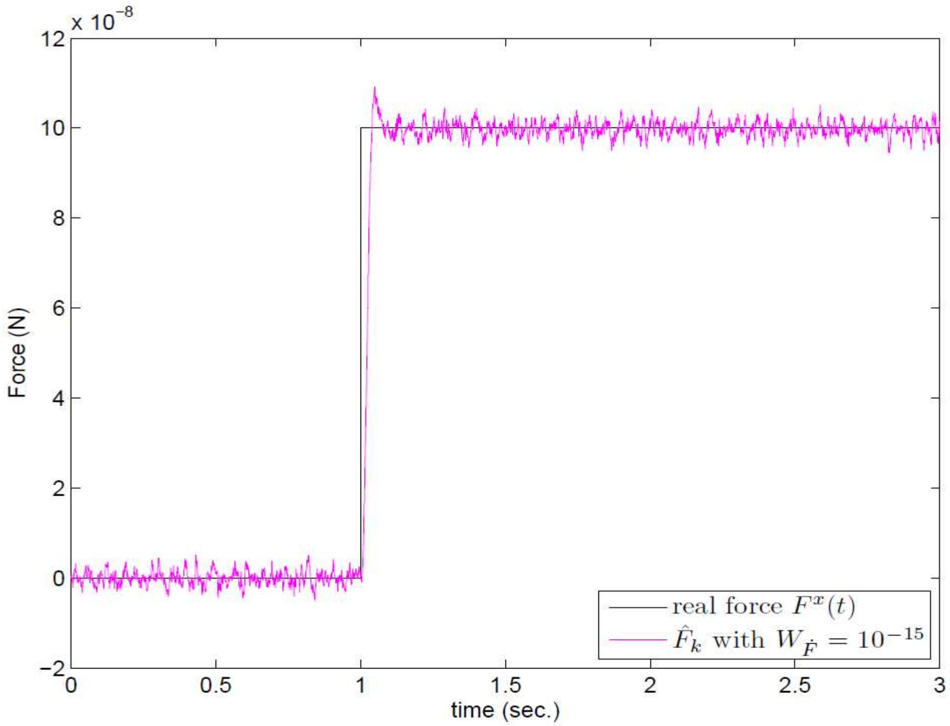
$$W_{\dot{F}}$$

Adjustment of the compromise between time response and variance of the estimated force

Influence of W_F parameter

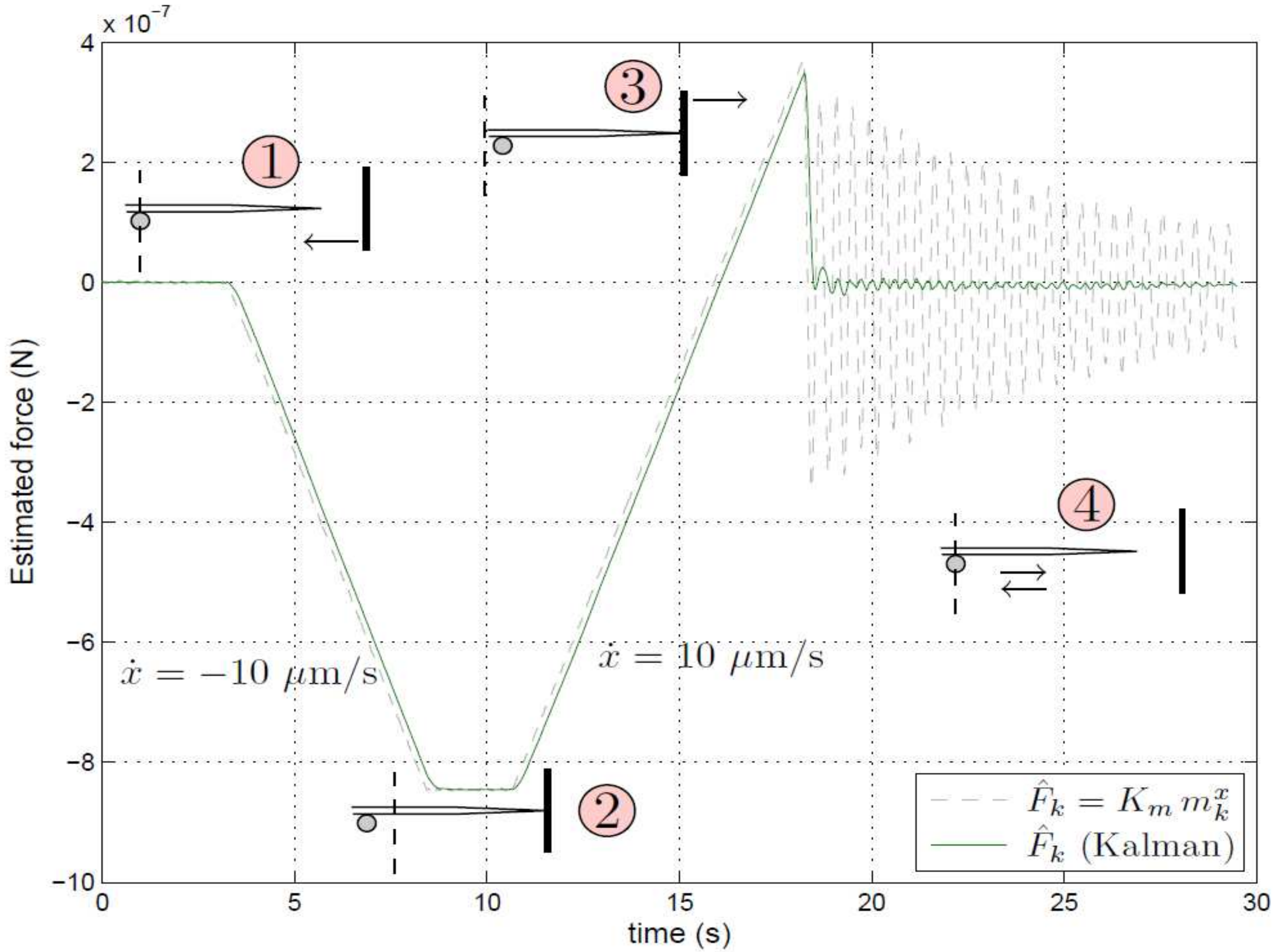


step-force estimation with $W_{\dot{F}} = 10^{-18}$ N^2/Hz



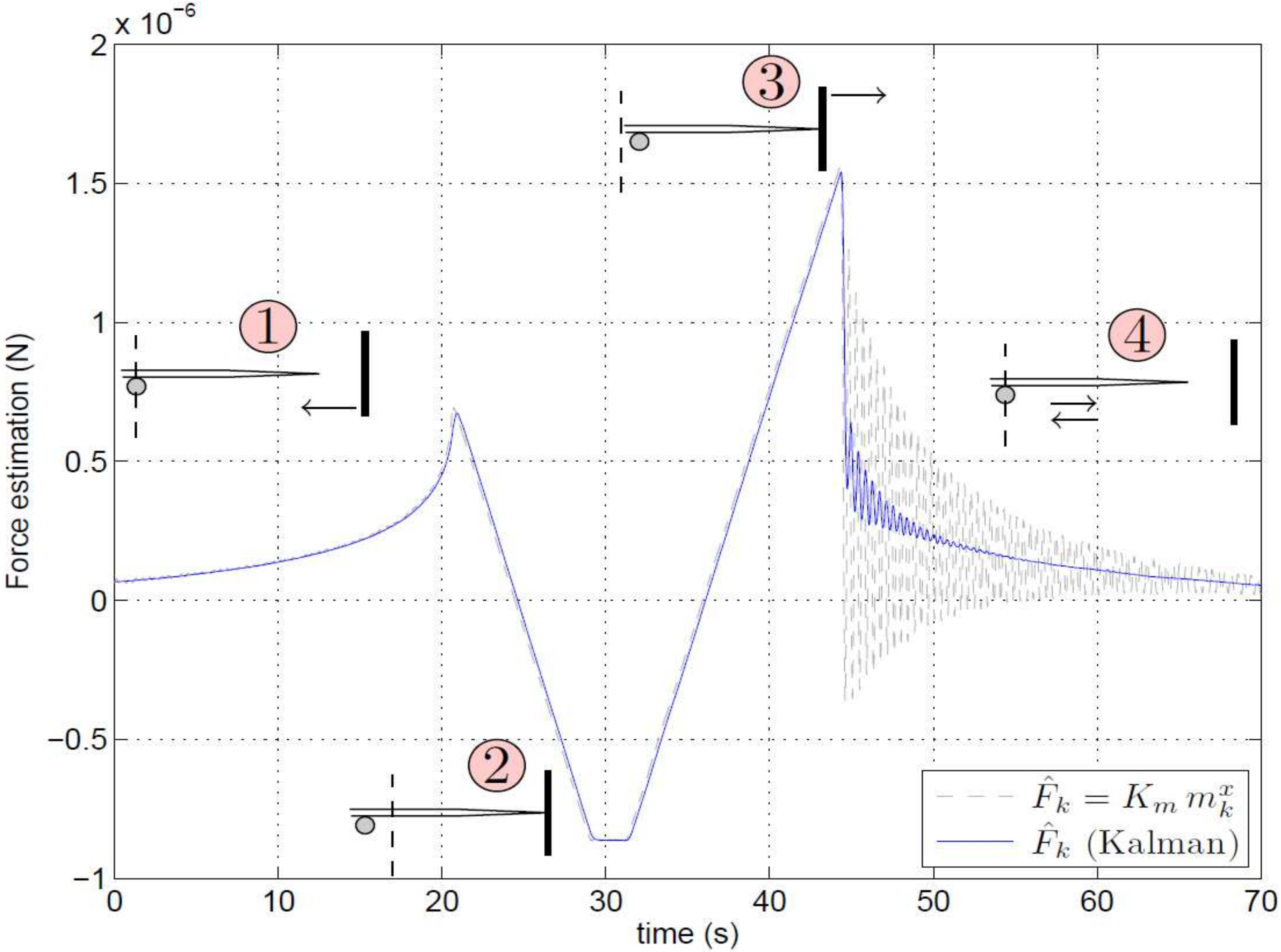
step-force estimation with $W_{\dot{F}} = 10^{-15}$ N^2/Hz

Pull-Off measurement



Experimental pull-off force measurement

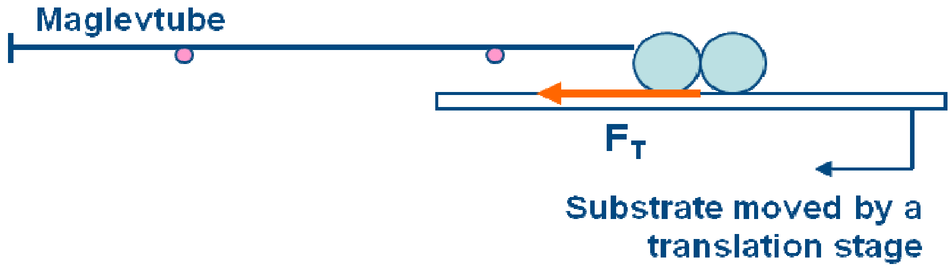
Pull-Off measurement with electrostatic force

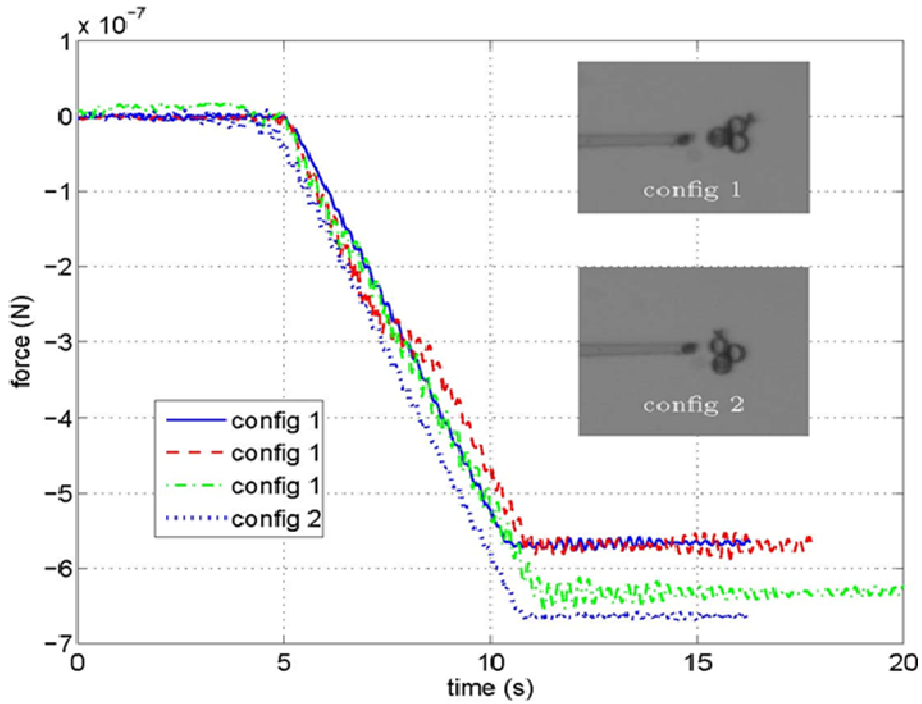


Friction force measurement

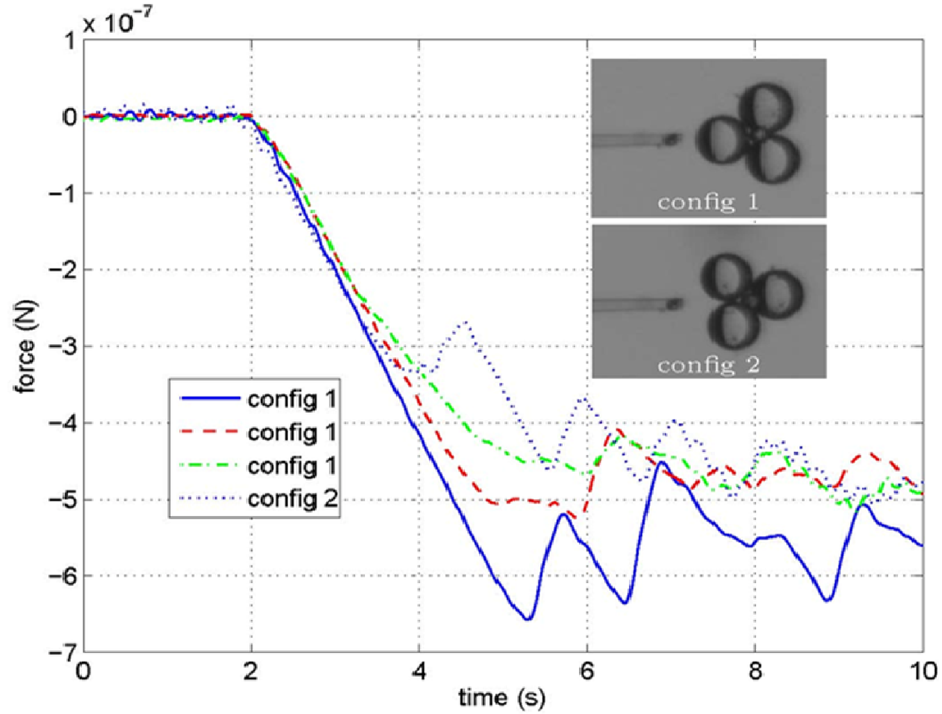
Friction study conditions :

µstructures with ∅40 µm and ∅100 µm spheres
Glass substrate cleaned in ethanol
20% to 30% relative humidity





∅40 µm spheres



∅100 µm spheres

Conclusion

- µ and nanoforce estimation based on a Kalman filter

- One parameter to set for adjusting the trade-off between resolution and response time.

- Process implemented in DSP on a experimental platform with good results.

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